Math 3310: Exam 1

1. Let A and B be finite sets such that $\#A = \#B$. Let $f : A \to B$ be injective. Prove that f is surjective.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $|f(x) - f(y)| \leq M|x - y|$ for all $x, y \in \mathbb{R}$, where $M \in \mathbb{R}$ is a positive, non-zero fixed constant. Prove that f is continuous.

3. Let $f : X \to Y$ be a function. Suppose that for **any** $\alpha : \{1\} \to X$ and $\beta : \{1\} \to X$, we have
$$f \circ \alpha = f \circ \beta \implies \alpha = \beta.$$
Prove that f is injective.

4. The *power set* of a set A, denoted $\mathcal{P}(A)$, is defined as the set of all subsets of A. For example, if $A = \{a, b\}$, $\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Prove that, for any function $f : X \to Y$, there is an induced function $\mathcal{P}(f) : \mathcal{P}(Y) \to \mathcal{P}(X)$ such that, if $g : Y \to Z$, then $\mathcal{P}(g \circ f) = \mathcal{P}(f) \circ \mathcal{P}(g)$, where $\mathcal{P}(g \circ f) : \mathcal{P}(Z) \to \mathcal{P}(X)$.

5. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \frac{1}{\cos(x)}$. Is f continuous? If yes, prove it. If not, explain why. You must use the ϵ/δ-definition of continuity in either case.