

Exam 2

1. Consider

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^4}{x^2 + y^2}.$$

Compute the limit along the two lines $y = 0$ and $y = mx$.

2. Consider the velocity vector $\vec{v}(t) = e^t \vec{i} - \sin(t) \vec{j} + \cos(t) \vec{k}$, with initial position $\vec{r}(0) = \vec{i} + \vec{j} + \vec{k}$. Compute $\vec{r}(t)$, the position vector.
3. Let $F(x, y) = \sin(x^2 y^2)$, where $x = \sin(u) + \cos(v)$ and $y = e^{u+v}$. Use the chain rule (substitution will earn zero credit) to find $\frac{\partial F}{\partial v}$.
4. Let $\vec{r}(t) = 8t \vec{i} + 3 \sin(t) \vec{j} + 3 \cos(t) \vec{k}$. Find an expression for the unit tangent vectors $\mathbf{T}(t)$.
5. Find the maximum rate of change of $f(x, y) = e^{-xy}$ at $(1, 1)$ and the direction in which it occurs.

Bonus Questions:

6. Given $(x + y)^2 + \sin(x + y) = y$, use the Implicit Function Theorem to find $\frac{dy}{dx}$.
7. Find the tangent plane to $z = x^2 + y^2$ at $(x, y) = (1, 2)$.