Math 3310: Final

1. Let (Y, τ) be a nonempty topological space. Show that there exists a nonempty topological space (X, τ') such that every function $f : X \to Y$ is continuous.

2. Let A, B be sets, and let $f : A \to B$ be a surjective function. Consider the relation on A defined as $x \sim y$ if and only if $f(x) = f(y)$. Prove that there exists a bijection from $A/\sim$ to B.

3. Let G be a group. The *order* of an element $g \in G$ is the smallest natural number k such that $g^k = e$. Prove that if G is a group in which every element has order at most 2, then G is abelian.

4. Let (X, τ) and (Y, τ') be topological spaces. A subset $A \subset X$ is *closed* if A^c is open. Prove that a function $f : X \to Y$ is continuous if and only if the preimage of any closed set is closed.

5. Let A, B, and C be sets. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

6. Let A, B be sets, and let $f : A \to B$ be surjective. Prove that if β_1, β_2 are maps from B to a set Z, then $\beta_1 \circ f = \beta_2 \circ f$ implies $\beta_1 = \beta_2$.

7. Let $f : (\mathbb{R}_{\geq 0}, |\cdot|) \to (\mathbb{R}_{\geq 0}, |\cdot|)$ be given by $f(x) = \sqrt{x}$. Prove, using the ϵ-δ definition of continuity, that f is continuous. Recall $\mathbb{R}_{\geq 0}$ was defined in class as
   $\mathbb{R}_{\geq 0} := \{x \in \mathbb{R} : x \geq 0\}$.